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## MODEL OF STEADY ISOTROPIC FLOW WITH POLYMER ADDITIVES

V. A. Sosinovich and V. A. Tsyganov

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The results of numerically solving the equation for the function describing the length scale distribution of the turbulent velocity fluctuation energy in a steady isotropic flow of dilute linear high-molecular polymer solution are presented.

### Introduction

In [1] a closed equation was derived for the function  $P_t(r)$  describing the length scale distribution of the turbulent velocity fluctuation energy in an isotropic flow of dilute high-molecular polymer solution. In this equation (cf. (11) in [1]) what is new is the closed, i.e., fully expressed in terms of the unknown function  $P_t(r)$  and the solution parameters, form of the term  $I(r, t)$ , which takes into account the effect of the polymer additives on the structure of the turbulent flow. In deriving the expression for  $I(r, t)$  the mechanism of interaction of the polymer molecules and the turbulence described in [2, 3] was used. According to this mechanism, the viscosity of the dilute polymer solution is selective with respect to the nature of the fluid motion: it is not affected by rotational motion and increases sharply when the motion involves tension.

### Formulation of the Steady-State Problem

With the expression for  $I(r, t)$ , represented by Eqs. (21)-(26) in [1], Eq. (11) of [1] can be solved numerically, given the initial form of the function  $P_0(r)$  and the parameters  $\nu$ ,  $\nu_p$ ,  $\varepsilon$ ,  $L$ , and  $T$ . In this article we will consider only the steady-state case, when the function  $P_t(r)$  does not depend on time, and the equilibrium in the flow is maintained by the balance between energy injection and dissipation. As will be seen from what follows, in the steady-state case the action of the polymer additives on the internal structure of the turbulent flow can be most graphically demonstrated.

In Eq. (11) [1] we pass to the limit as  $t \rightarrow \infty$ . Using the equalities

$$\begin{aligned} \lim_{t \rightarrow \infty} P_t(r) &= P_\infty(r) \equiv P(r), \\ \lim_{t \rightarrow \infty} \frac{\partial P_t(r)}{\partial t} &= 0, \quad \lim_{t \rightarrow \infty} I(r, t) \equiv I(r), \end{aligned} \quad (1)$$

we can write the equation for  $P(r)$  in the form:

$$\frac{d}{dr} \left\{ \left[ 2\nu + 2\nu_p \int_0^r \sqrt{r'P(r')} dr' \right] \left( \frac{d}{dr} + \frac{4}{r} \right) P(r) + \nu_p I(r) \right\} = \frac{4}{3} \varepsilon \frac{r}{L^2} \exp\left(-\frac{r^2}{L^2}\right). \quad (2)$$

Integrating this equation with respect to  $r$  from 0 to  $r$  and using Eq. (21) from [1], together with the equations

$$I(0) = 0, \quad \varepsilon = 15\nu \left. \frac{dP(r)}{dr} \right|_{r=0}, \quad (3)$$

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we obtain the form of the steady-state equation for  $P(r)$ :

$$\left[2\nu + 2\gamma \int_0^r \sqrt{r'P(r')} dr'\right] \left(\frac{d}{dr} + \frac{4}{r}\right) P(r) - \nu_p G(r) \frac{P(r)}{r} = \frac{2}{3} \varepsilon \exp\left(-\frac{r^2}{L^2}\right). \quad (4)$$

We will go over to dimensionless notation in Eq. (4). In the steady-state case as the characteristic length it is convenient to choose the scale  $L$  on which energy is pumped into the flow. As the characteristic velocity we will take  $\sqrt{\frac{2}{3}q}$ , where  $q$  is the energy contained in unit volume of the turbulent flow in the steady state:

$$\frac{2}{3}q = \int_0^\infty P(r) dr. \quad (5)$$

Then the characteristic values of the function  $P(r)$ , the injection rate, the dissipation rate, which in the steady state is equal to it, and the turbulent fluctuation time scale will be:

$$P_c = \frac{2}{3}q/L, \quad \varepsilon_c = \frac{(2/3q)^{3/2}}{L}, \quad T_c = \frac{L}{\sqrt{\frac{2}{3}q}}. \quad (6)$$

The Reynolds number can be expressed as

$$Re = \frac{\sqrt{\frac{2}{3}q}L}{\nu}. \quad (7)$$

Equation (4) now takes the dimensionless form:

$$2 \left[ \frac{1}{Re} + \gamma \int_0^\rho \sqrt{\rho' \hat{P}(\rho')} d\rho' \right] \left( \frac{d}{d\rho} + \frac{4}{\rho} \right) \hat{P}(\rho) - \frac{n}{Re} \hat{G}(\rho) \frac{\hat{P}(\rho)}{\rho} = \frac{2}{3} \hat{\varepsilon} \exp(-\rho^2). \quad (8)$$

Here,  $\rho = r/L$ ; the symbols  $\hat{\phantom{x}}$  denote nondimensionalized quantities:

$$\hat{P}(\rho) = \frac{P(r/L)}{P_c}; \quad \hat{\varepsilon} = \frac{\varepsilon}{\varepsilon_c}. \quad (9)$$

The number  $n$  is determined by the ratio of the viscosity coefficients of the solution  $\nu_p$  at the points at which the molecules are stretched and the viscosity of the solvent:

$$n = \frac{\nu_p}{\nu}. \quad (10)$$

This number is proportional to the polymer molecule concentration  $c_p$ :

$$n \sim c_p N, \quad (11)$$

where  $N$  may be assumed to be approximately equal to the number of monomer units in the polymer molecule [2, 3].

In dimensionless quantities the property (5) of the function  $P(r)$  can be written in the form:

$$\int_0^\infty \hat{P}(\rho) d\rho = 1. \quad (12)$$

In dimensionless variables the quantities  $\omega_1$  and  $\omega_2$  will have the form:

$$\omega_{(1)} = \frac{1}{3\hat{T}^2 [\hat{P}'(0) \pm \hat{P}'(\rho)]}. \quad (13)$$

Here

$$\hat{T} = \frac{T}{T_c} = \frac{T}{L} \sqrt{\frac{2}{3}q} \quad (14)$$

represents the ratio of the characteristic relaxation times of the polymer molecule and the turbulence. The second of Eqs. (3) can be written in the dimensionless form:

$$\hat{P}'(0) = \frac{Re \hat{\epsilon}}{15} \quad (15)$$

It can also be obtained from Eq. (8) by passing to the limit as  $\rho \rightarrow \infty$ . From this equation it is clear that the quantity  $\hat{\epsilon}$  in (8) is not a free parameter, but is linked to  $Re$  through the quantity  $\hat{P}'(0)$ , which is the result of the solution of Eq. (8) and the integral condition (12). Therefore, the problem of calculating the function  $\hat{P}(\rho)$  can be posed as follows: for a given  $Re$  to choose a value of  $\hat{\epsilon}$  such that the function  $\hat{P}(\rho)$  found as a result of solving Eq. (8) satisfies condition (12). The solution must be obtained for values of  $n$  on the interval  $[0, \infty]$ . The value  $n = 0$  corresponds to the absence of polymer:  $c_p = 0$ . Large values of  $n$  correspond to polymer concentrations  $c_p$  at which saturation sets in.

When a change in the Reynolds number is achieved at the expense of a change in the fluctuating velocity at a fixed length scale  $L$ , the expression for  $\omega_{(2)}^{(1)}$  can be represented in the form:

$$\omega_{(2)}^{(1)} = \frac{5}{\tau^2 Re^3 \hat{\epsilon} (1 \pm \hat{Y})}, \quad (16)$$

where  $\tau$  is the ratio of the relaxation time of the polymer molecule to the characteristic viscous dissipation time on the length scale  $L$ :

$$\tau = \frac{T}{\tau_T}, \quad (17)$$

where

$$\tau_T = \frac{L^2}{\nu}. \quad (18)$$

The quantity  $\tau_T$  is related to the time scale of the turbulent fluctuations  $T_c$  by the expression:

$$\tau_T = T_c Re. \quad (19)$$

In solving Eqs. (8) and (12) it is necessary to know the function  $\hat{G}$  at each value of  $\rho$  for the chosen values of the Reynolds number and the relaxation time and concentration of the polymer molecules. As may be seen from (22), (23) in [1], which determine the function  $\hat{G}$ , the value of this function does not depend directly on the variable  $\rho$  and on the polymer molecule concentration. This dependence is realized implicitly through the variable  $\hat{Y}$ :

$$\hat{Y} = \frac{\hat{P}'(\rho)}{\hat{P}'(0)}, \quad (20)$$

whose value is determined by the structure of the acceleration field. Having chosen specific values of the quantities  $T$ ,  $\nu$ , and  $L$  ( $T = 10^{-3} \cdot \text{sec}$ ,  $\nu = 10^{-6} \text{ m}^2 \cdot \text{sec}^{-1}$ ,  $L = 10^{-3} \text{ m}$ ) and having thereby determined the value of  $\tau$ , we can calculate the function  $\hat{G}$  in advance over the entire range of possible values of the variables  $\hat{Y}$  and  $\hat{\epsilon}$  and for various Reynolds numbers. In the process of numerically solving the Eqs. (8), (12) it is possible to employ the calculated values of  $\hat{G}$ , choosing those values which correspond to the current values of  $\hat{Y}$  and  $\hat{\epsilon}$  and the  $Re$  number for the calculation variant in question.

### Results of Numerically Solving the Equation for $\hat{P}(\rho)$

The form of the function  $\hat{P}(\rho)$  obtained in solving Eqs. (8), (12) is shown in Fig. 1. A comparison of the curves for different values of  $n$  (Fig. 1a) shows that as the polymer molecule concentration increases, the energy distribution is displaced towards larger length scales. This is because the polymer additives reduce the turbulent energy transfer over the length scale spectrum. It is also clear from Fig. 1a that at large values of  $n$  the curves are crowded together, demonstrating the saturation effect [2, 3], i.e., the reduced effectiveness at overly high concentrations. Curve 5 in Fig. 1a describes the velocity fluctuation energy distribution in the hypothetical situation in which there is absolutely no turbulent transfer. Comparison of this curve with the curves corresponding to turbulent flow at high polymer concentrations suggests that the polymer additives are not capable of completely suppressing turbulent transfer.

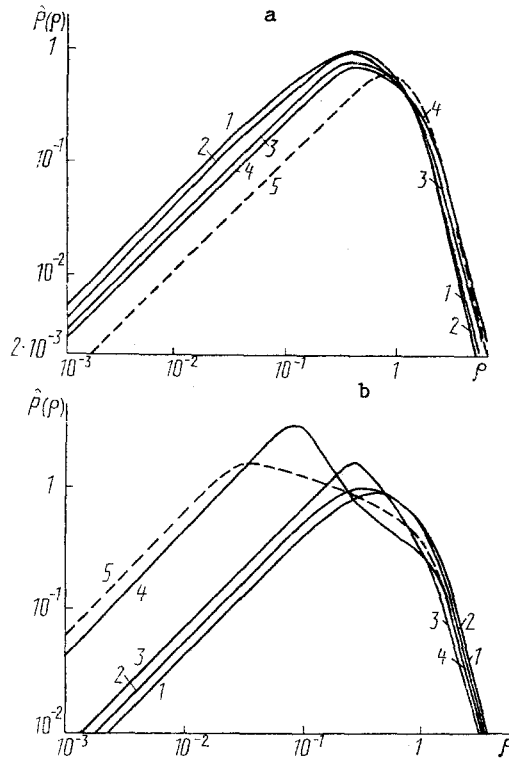


Fig. 1. Dependence of the function  $\hat{P}(\rho)$  on the polymer concentration (a) and the Reynolds number (b): (a)  $Re = 50$ , 1)  $n = 0$ ; 2) 1; 3)  $10^2$ ; 4)  $10^4$ ; 5)  $\gamma = 0$ ; (b)  $n = 1$ , 1)  $Re = 50$ ; 2) 100; 3) 200; 4) 1000; 5)  $n = 0$ ;  $Re = 1000$ .

It is clear from Fig. 1b that the shape of  $\hat{P}(\rho)$  depends on the Reynolds number in the same way, with and without polymer additives: as the  $Re$  number increases the turbulent fluctuation energy distribution becomes more small-scale. At high  $Re$  numbers it is possible to distinguish an inertial length scale range on which  $\hat{P}(\rho) \sim \rho^{-1/3}$ . The difference consists in that when  $n \neq 0$  the value of the function  $P(\rho)$  for small values of  $\rho$  is significantly smaller than when  $n = 0$  (this can be confirmed by comparing curves 4 and 5).

In Fig. 2a we have plotted the function

$$T(\rho) = 2\gamma \int_0^{\rho} \sqrt{\rho \hat{P}(\rho')} d\rho' \left( \frac{d}{d\rho} + \frac{4}{\rho} \right) P(\rho) \quad (21)$$

against the polymer molecule concentration. In Eq. (8) the function  $T(\rho)$  describes that part of the turbulent energy flux which is directed from large to small length scales. Clearly, this energy flux decreases with increase in the polymer concentration and the decrease is especially marked on the interval of small length scales. For large scales the opposite dependence on concentration is observed.

We note that when  $n \neq 0$  the term  $T(\rho)$  in (8) does not have a well-defined physical meaning. The same can also be said of the last term in (8), which describes the turbulent energy transfer from small to large scales. Only the sum of these two terms

$$S(\rho) = T(\rho) - \frac{n}{Re} \hat{G}(\rho) \frac{P(\rho)}{\rho} \quad (22)$$

has a clear physical content, describing the resultant turbulent energy flux. It is clear from Fig. 2b that the total energy flux in the small scale direction decreases strongly with increase in  $n$  and at high polymer concentrations is ensured only by the large-scale motions of the fluid. This directly confirms the previous conclusion to the effect that the dissipativeness of the turbulence is reduced by polymer additives.

To obtain a clearer demonstration of the effect of polymers on turbulence it is convenient to determine the efficiency  $\delta$  from the following expression:

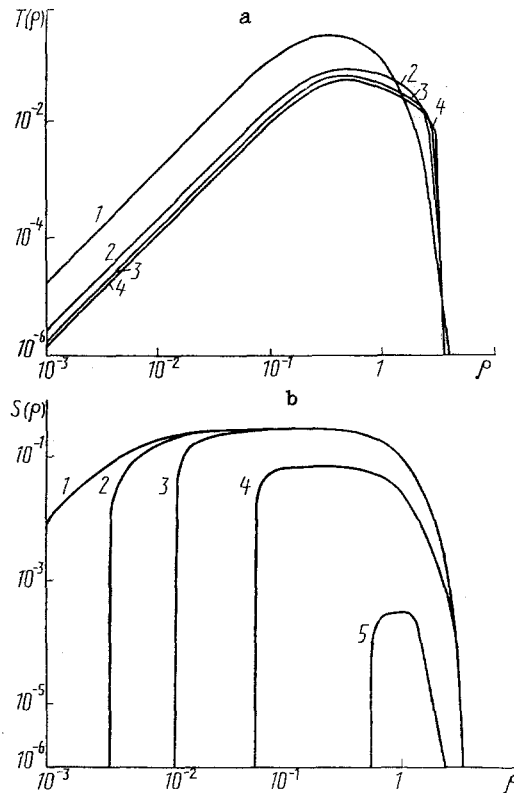


Fig. 2. Dependence of  $T(\rho)$  (a) and  $S(\rho)$  (b) on the polymer concentration,  $Re = 100$ : (a) 1)  $n = 0$ , 2)  $10^2$ , 3)  $10^3$ , 4)  $10^4$ ; (b) 1)  $n = 0$ , 2)  $0.1$ , 3)  $1.0$ , 4)  $10$ , 5)  $100$ .

$$\delta(n, Re) = \frac{\hat{\varepsilon}(0, Re) - \hat{\varepsilon}(n, Re)}{\hat{\varepsilon}(0, Re)} = 1 - \frac{\hat{\varepsilon}(n, Re)}{\hat{\varepsilon}(0, Re)} \quad (23)$$

Figure 3 shows the dependence of  $\delta$  on the polymer concentration and the Reynolds number. It is clear from Fig. 3a that the efficiency corresponding to the concentrations at which saturation sets in increases with  $Re$ . At large  $Re$  saturation sets in fairly sharply. At lower values of  $Re$  the concentration dependence is weaker. This behavior is consistent with the experimental data on drag reduction [2-4]. The threshold nature of the dependence of the effect on the Reynolds number is apparent from Fig. 3b. For all concentrations the increase in efficiency begins at  $Re \approx 30$ .

Figure 3b illustrates another important feature of the interaction between polymer molecules and turbulence: for each concentration value there is an optimum Reynolds number for which the efficiency has a maximum. This behavior can be explained as follows. As the Reynolds number increases, the time scale of the turbulent fluctuations decreases and becomes comparable with the characteristic relaxation of the polymer molecules. As a result, there appear in the flow ever more points at which the molecule stretching criterion is satisfied. For fairly high values of  $Re$  this criterion is satisfied almost everywhere. A further increase in  $Re$  is less effective, since all the molecules in the flow are stretched. As may be seen from Fig. 1 in [1], as  $Re$  increases the function  $\bar{G}$  begins to take its maximum value even at small values of  $\varepsilon$ . At the same time, an increase in  $Re$  implies a further increase in the rate of inertial energy transfer to the small scales. The relation between the two terms in the expression for the resultant energy flux (see (22)) begins to change in favor of the first term, as a result of which the dissipativeness of the flow increases.

This dependence of the efficiency on the Reynolds number, which has also been confirmed experimentally [4], is related to the "viscous" origin of the polymer term in (22) and directly confirms the validity of the interaction mechanism on which the theory is based.

From the resonance nature of the dependence of the efficiency on  $Re$  there follows the possibility of choosing the optimum hydrodynamic regime for a given polymer solution.

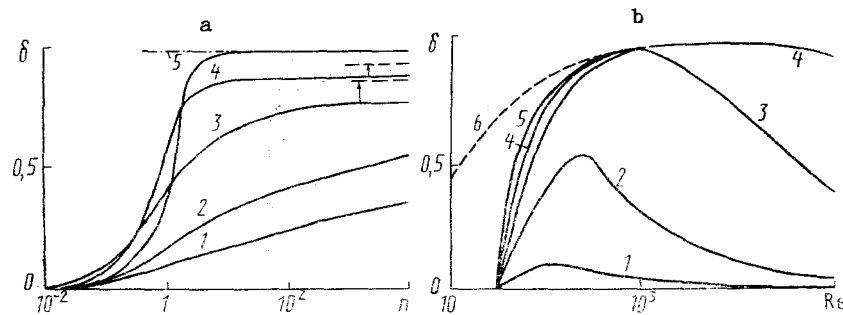


Fig. 3. Efficiency as a function of concentration for various Reynolds numbers (a) and as a function of the Reynolds number for various concentrations (b): (a) 1)  $Re = 40$ ; 2) 50; 3) 100; 4) 200; 5) 1000; broken lines) maximum efficiency levels for a given Reynolds number; (b) 1)  $n = 0.1$ ; 2) 1.0; 3) 10; 4) 100; 5)  $10^4$ ; 6) the maximum efficiency curve ( $\delta = 1 - \epsilon/\epsilon_{n=0}$ ).

Another important feature of the problem is associated with the possibility of choosing polymer molecules with different values of the characteristic relaxation time  $T$ . As may be seen from (16) and (17),  $T$  enters into the determination of the functions  $\omega_{(2)}$  and hence  $\hat{G}$ . Thus, the values of  $T$  strongly influence the function  $\hat{G}$ .

Figure 4 shows the efficiency as a function of the Reynolds number and the polymer concentration for various values of  $T$ . It is clear from Fig. 4a that with decrease in the polymer molecule relaxation time  $T$  the beginning of the effect is displaced towards higher Reynolds numbers. This is natural since a decrease in  $T$  means that the polymer molecules in the flow are stiffer and stretch at higher values of the Reynolds number.

It can be seen from Fig. 4b that a decrease in  $T$  leads to a diminution of the effect at any concentration. As  $T$  tends to zero, the effect disappears completely, since in this case the conditions of stretching of the molecules in the turbulent flow cannot be realized.

Figure 5 shows the efficiency as a function of the relaxation time for various values of the Reynolds number. It is noteworthy that the efficiency increases, reaches a maximum and does not decrease with increase in  $T$  for any value of the Reynolds number. From this we may conclude that polymers with a large relaxation time are more efficient. From this figure it is also clear that for polymers with small  $T$  the regime with high Reynolds numbers is optimal, while for polymers with a larger relaxation time  $T$  the regime with lower Reynolds numbers is more favorable.

We note that  $T$  enters into the determination of the function  $\hat{G}$  through the dimensionless complex  $\tau = T/\tau_T$ , where  $\tau_T = L^2/\nu$  (Eqs. (16)-(18)). This means that all the effects associated with a change in  $T$  can be achieved by varying the length scale  $L$ . By choosing a suitable characteristic velocity  $\sqrt{\frac{2}{3}q}$  it is possible to leave the Reynolds number unchanged.

This explains the experimentally observed diameter effect, namely that the quantitative results associated with the action of polymers are different in pipes of different diameter for the same Reynolds number [2-4].

The second diameter effect, associated with the fact that a degraded polymer solution displays the original efficiency in pipes of smaller diameter [2], also finds a natural explanation within the framework of the present model. Thus, the value of  $\tau$ , reduced as a result of degradation, can be restored by a decrease in the length scale  $L$ .

We also note that if we plot all the relations  $\delta(Re, n)$ , calculated for any values of the polymer molecule concentration and any relaxation time  $T$ , then they do not rise above a certain conventional line, which passes below the line characterizing the relation  $\delta(Re)$  in the absence of turbulent transfer. This line is the analog of the experimentally established Virk asymptote [2].

## Conclusions

In [1] an equation was derived for the function  $P_t(r)$  describing the length scale distribution of the turbulent velocity fluctuation energy in a flow of dilute linear high-molecular/polymer solution. The derivation was based on the assumption that in the turbulent

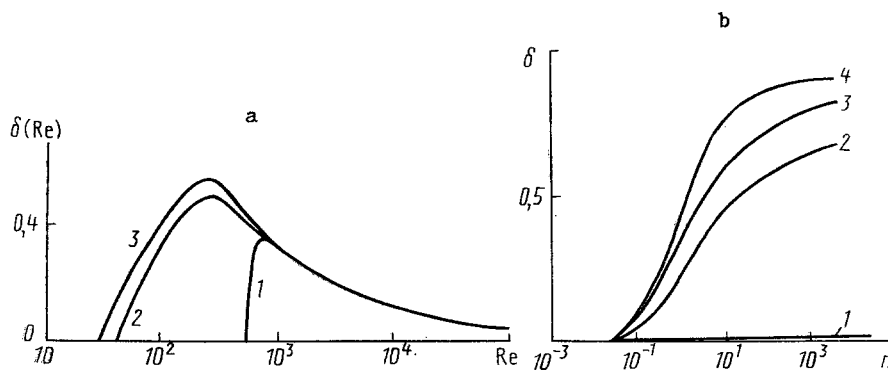


Fig. 4. Efficiency as a function of Reynolds number (a) and polymer concentration (b) for various values of  $T$ ,  $Re = 100$ : (a) 1)  $T = 0.3 \cdot 10^{-3}$  sec; 2)  $0.707 \cdot 10^{-3}$ ; 3)  $10^{-3}$ ; (b) 1)  $T = 0$ ; 2)  $0.5 \cdot 10^{-3}$  sec; 3)  $0.707 \cdot 10^{-3}$ ; 4)  $10^{-3}$ .

flow the polymer molecules are subjected to intense stretching at points in the flow at which the stretching criterion is satisfied, so that at those points the viscosity of the fluid increases.

An analysis of the equation obtained shows that the presence of the polymer makes the turbulent flow less dissipative.

In this study the steady-state form of the equation derived in [1] is solved numerically. It is established that the addition of a polymer leads to substantial reorganization of the spectral structure of the turbulent flow. As the polymer concentration increases, the function  $\hat{P}(\rho)$  decreases on the small length scale interval and increases at large length scales. The form of the function  $\hat{P}(\rho)$  changes only slightly. On the inertial length scale range the usual law of variation of this function  $\hat{P}(\rho) \sim \rho^{-1/3}$  is preserved. The bunching of the curves with increase in concentration confirms the existence of a saturation effect. The function  $S(\rho)$  describing the total turbulent energy transfer over the length scale spectrum varies strongly. With increase in concentration this function is nonzero only in the region of the length macroscale of the turbulent fluctuations. The form of this function at high values of the polymer molecule concentration directly demonstrates the decrease in dissipativeness.

It is shown that the efficiency of the polymer admixture will depend on the Reynolds number and for each concentration of a given polymer there is an optimum value of  $Re$  for which the efficiency is maximal.

The threshold nature of the Toms effect, observed experimentally, is confirmed. For a polymer with a specific value of the relaxation time  $T$  the effect begins at the same Reynolds number for all concentrations. In the case of polyethylene oxide, for which  $T \sim 1 \cdot 10^{-3}$  sec, the threshold Reynolds number is equal to 30. It is shown that as  $T$  decreases, the value of the threshold Reynolds number increases strongly.

An analysis of the solution shows that the efficiency of the polymers  $\delta$  reaches maximum values as  $T$  increases, i.e., "soft" polymer molecules (molecules with a high molecular weight) are the most efficient. When  $T < 5 \cdot 10^{-3}$  sec the efficiency increases as  $Re$  increases, and when  $T > 11 \cdot 10^{-3}$  sec it decreases. At intermediate values of  $T$  the dependence of the efficiency on  $Re$  is irregular.

The results obtained can be useful for choosing the optimum hydrodynamic regime in working with specific polymers and for choosing a suitable polymer solution if the hydrodynamic regime is determined by other factors.

It is shown that the polymer molecules are not capable of completely suppressing turbulent transfer. There is a certain minimum  $\delta(Re)$  relation which passes below that obtained in the absence of turbulent friction and is an analog of the experimentally confirmed Virk asymptote.

The model provides a perfectly natural explanation of the experimentally observed diameter effect and the so-called second diameter effect associated with the degradation of the polymer solution.

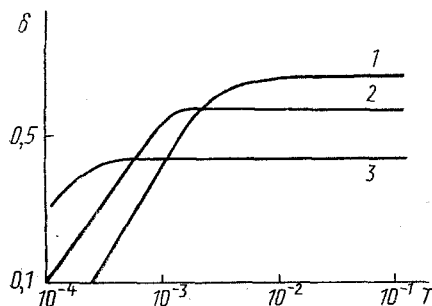


Fig. 5. Efficiency as a function of  $T$  for various values of  $Re$ ,  $n = 1$ : 1)  $Re = 100$ ; 2) 200; 3) 500.

From the above it is clear that our results are in good qualitative agreement with the experimentally observed facts relating to the effect of polymer admixtures on turbulence [2, 3]. It is also possible to note a certain quantitative correspondence. An analysis of the dependence of the efficiency on the polymer concentration shows that the concentrations range from the appearance of the effect to the state of saturation spans 2-3 orders. This is consistent with the analogous experimental curves [2-4].

At  $Re \sim 100$  (which corresponds to values of the external Reynolds number  $Re \sim 10^5$ ) the maximum efficiency reaches 80%. This is also in agreement with the experimental data [2-4].

In general, the theoretical model developed provides a good basis for solving problems of practical importance, in particular problems associated with the optimization of the process of turbulent mixing of polymer solutions in water.

In these circumstances it is necessary to be able to solve the problem of the evolution of the velocity field and the scalar field of the polymer admixture, which are interrelated.

#### NOTATION

$P_t(r)$ , function describing the length scale distribution of the turbulent fluctuation energy;  $T$ , characteristic relaxation time of the polymer molecule;  $\nu$ , viscosity coefficient of the solvent;  $\nu_p$ , longitudinal viscosity coefficient of the solution;  $B(r, t)$ , longitudinal correlation function of the velocity field;  $q(t)$ , average velocity field turbulent fluctuation energy;  $f_i \equiv f_i(x, t)$ , random force vector field;  $I(r, t)$ , function describing the effect of the polymer on the structure of the function  $P_t(r)$ ;  $L_0$ , characteristic length scale of the initial velocity field;  $T_0$ , characteristic time of the turbulent velocity fluctuations at  $t = 0$ ;  $\hat{\epsilon}$ , nondimensionalized turbulent energy dissipation rate;  $\hat{\gamma} = \frac{P'(\rho)}{P'(0)}$ ,  $\tau = \frac{T}{\tau_r}$ ,  $\tau_r = L^2/\nu$ , the characteristic viscous dissipation time at the length scale  $L$ .

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